Budget Induced Strategic Bidding in Multiunit Online Auctions

Completed Research Paper

Introduction

The online auction has been a popular and representative model in online e-commerce over the past decade. According to a survey of National Consumers League (NCL), since eBay launched its online auction platform in 1995, nearly one-third of the online population in the U.S. have participated in an online auction. Compared with traditional auctions, the arrival process for online auctions is more random and dynamic. The stochastic arrival process, coupled with a complex decision environment for bidders that include both simultaneous and sequential auctions, engenders interesting bidding behaviors in online auctions. One of the common-observed phenomena in online ascending auctions is *jump bidding*, defined as submitting bids higher than required by the auctioneer (Zheng, 2012). Jump bidding is especially relevant for online auctions because of the uncertainties in both the number of bidders and their arrival process in online environments. For practitioners, jump bidding has "important revenue effects" (Avery, 1998). In some cases, jump bidding may reduce the seller's expected revenue (e.g. when the jump bidding deters other bidders from participating), though some argue that jump bidding can potentially increase the seller's revenue and bidders' utility because it may improve efficiency. Regardless the direction of impact, understanding the underlying mechanisms of jump bidding is crucial for sellers and buyers in online auctions.

In academic research, several reasons for jump bidding have been identified (Avery, 1998; Easley & Tenorio, 2004; Gunderson & Wang, 1998; Isaac, Salmon, & Zillante, 2007) These include the signaling of an aggressive strategy (Avery, 1998), saving bidding cost (Easley & Tenorio, 2004), impatience (Isaac et al., 2007), and irrationality (Gunderson & Wang, 1998). Many of the existing explanations, however, do not carry over to an environment where a software agent place bids on behalf of human buyers. For example, participation costs, impatience, and irrationality may not explain the jump-bidding behavior in agent-based bidding, which is increasingly common on online auction platforms. Even when agent-based bidding is used, there are still strategic reasons for jump bidding. However, little attention has been paid to the latter kind of reasons for jump bidding.

In our study, we address the aforementioned gap by proposing a novel explanation for jump bidding based on strategic behaviors under budget constraints. In real-world auctions, many bidders have budget constraints, which may force them to drop out an auction even when they have a significant positive value for an item. This means that budget-constrained bidders would strongly prefer winning than losing (which forgoes the positive value for the item). Jump bidding let such bidders enter a high bid early, thus increasing their likelihood to win. This may cause bidders to pay a higher final price, but when bidders have a strong preference for winning, they may risk a higher final price in exchange for a higher winning chance.

This research has two goals. First, we hope to theoretically establish the condition under which such jump bidding behaviors occur. Second, we hope to further understand what factors play a key role in driving budget-induced jump bidding. For the first goal, we conduct a theoretical analysis. For the second, we rely on simulation.

The context of this study is multi-unit ascending online auctions in which multiple bidders with unit-demand compete for multiple identical items in an ascending auction. Because budget-induced jump bidding tends to happen near the end of a bidder's bidding cycle, we will focus on a bidder's strategy at *his margin*, i.e., when the current price is one bid increment away from the maximum bid the bidder can submit within his budget constraint. In our theoretical model, we analyze two bidding strategies when the auction arrives at a bidder's margin: a *participatory* strategy in which the bidder places the minimum required bid and a *jump* strategy in which the bidder bids one increment more than the required bid level, or his maximum bid. We derive the condition for the bidder to use the jump strategy. Based on this theoretical analysis, we characterize the optimal hybrid strategy (between *participatory* and *jump*) for an at-the-margin, budget-constrained bidder and call it strategic-at-margin (SAM) bidding.

We then use the simulation method to validate our proposed SAM bidding by benchmarking it against a pure participatory strategy and a pure jump strategy. Our simulation also explores how the budget constraint and the bid increment jointly influence jump bidding at the margin.

Related Literature

This study is mainly related to three research streams, discussed in turn below.

Bidding Strategies in Multiunit Online Auctions

The bidding strategies in multiunit online auctions have been classified into three broad categories(Bapna, Goes, & Gupta, 2001): evaluatory, participatory and opportunistic. The evaluatory strategy refers to placing only the highest feasible bid in the auctions which is much higher than the minimum required bid at that time. The "participatory" strategy (also known as the "pedestrian" strategy) means to participate in the auction on an ongoing basis to place a current minimum required bid. They continue this process until the minimum required bid is higher than their valuation. The opportunistic strategy is to place a bid just before the end of the auction to look for bargains.

Some theoretical work establishes that the participatory strategy is also optimal for multi-unit ascending auction in the absence of bidding costs(Rothkopf & Harstad, 1994a, 1994b). Bidders using a participatory strategy can win the auction at the minimum price with a significant probability (Bapna et al., 2001). In addition, some online auction platform develops a software agent to submit bids for the bidders (Easley & Tenorio, 2004), resulting in a similar behavior as the participatory strategy.

Jump Bidding

This research is related to the literature on jump bidding. Jump bidding is a strategy of bidding more than the minimum required level during the bidding process. The reasons why jump bidding occurs are analyzed and discussed in the theoretical and empirical literature. One popular explanation is that bidders use jump bids to signal to other competitors their values and intimidate them into dropping out of the auction earlier (Avery, 1998; Gunderson & Wang, 1998). Another explanation is that jump bidders are behaving irrationally (Gunderson & Wang, 1998). However, a field study shows that signaling and irrationality cannot fully account for the existing jump bidding behavior (Isaac et al., 2007). To account for the observed jump bidding behavior, (Isaac et al., 2007) proposes additional mechanisms that jump bidding may arise because of bidders' impatience and strategic concerns. Easley and Tenorio (2004) provide a reason for jump bidding in online live auctions based on participation costs (both opportunity cost of time and time and effort taken to place a new bid each time) and uncertainty about the future. Our budget-induced explanation falls into the category of strategic jump bidding, which is deemed as "commonly overlooked" and "least understood" (Isaac et al., 2007). (Isaac et al., 2007) gives examples of "notch" bidding as one type of strategic jump bidding where one bidder can catch other bidders inside the notch of the increment to keep them from bidding again. Our explanation of strategic jump bidding adds the consideration of bidders' budget, which has not been formally considered in the jump bidding literature.

We note that signal-based jump bidding is more likely to happen early in an auction for deterring potential bidders from entering the auction. Similarly, jump bidding for saving participation costs also imply early jump bids. In contrast, strategic jump bidding including (Isaac et al., 2007)'s "notch" bidding and our budget-induced jump bidding tend to happen at the end of a bidder's biding activities. We note that jump bidding behavior studied in this paper differs from another well-known phenomenon of "late bidding (or snipping)" which refers to bids tend to concentrate near the auction's closing time (e.g., Bajari, Hortaçsu, & Hortacsu, 2003). Jump bidding in our context happens towards the end of a bidder's bidding cycle, which may or may not be near the closing time of the auction.

Auction Bidding with Budget Constraints

This research is related to research on budget constraints in auctions. Existing literature on the budget constraint in static auctions shows that budget constraints can impact bidders' aggressiveness (Che & Gale, 1996; Kotowski, 2018) and depress their bid levels (Kotowski, 2018). Most existing research about dynamic bidding under budget constraints focuses on specific contexts, such as sponsored search auctions hosted by search engines (Gummadi, Key, & Proutiere, 2011). There is little research on how budget constraints affect jump bidding.

Model

Our research context is an ascending multi-unit online auction (also known as the Yankee auction). In such an auction, there are Q identical units to be sold to N $(Q \leq N)$ buyers, each of whom demands at most one unit. The auction accepts increasingly higher bids until there is no new bid. The auction maintains a *provisional winning list* of size Q or smaller and a *bidding level* b, such that all provisional winning bids are equal to or greater than b, and only new bids of b or higher can enter the winning list. The winning ranked first by the bidding level, then by their arrival time (a later bid is ranked lower).

When the winning list is full (i.e. has a size of Q), the bidding level is the minimal bid among the winning list plus a *minimal bid increment* δ; any new bid will displace the last bidder in the winning list,. When the winning list is not full, the bidding level b is the minimal bid among the willing list. A new bid, in this case, will be appended to the winning list. At the end of the auction, i.e., when there is no new bid, bidders on the winning list are winners and they each pay their own prices.

We assume all bids must be multiples of the minimal bid increment δ. We denote O as the opening bid of the auction, which is usually set by the seller. We denote W as the *winning level* of the auction, which equals to the minimum bid in the final winning list.

All bidders' valuations are drawn from a common distribution, which has a cumulative density function of F. Denote V_i as the i-th bidder's valuation and B_i $(B_i \leq V_i)$ as the bidder's budget, which is also his *maximum feasible bid*. If the i-th bidder doesn't have a budget constraint, we let $B_i = V_i$. We denote $G_i = V_i - B_i$ as the gap between the bidder's budget and valuation, i.e. his *budget gap*. For notational simplicity, we omit the subscript i and use V, B, and G to denote the bidder i's valuation, budget, and budget gap, respectively. Clearly, a bidder doesn't have a chance to win the auction if his maximum feasible B is smaller than the winning level W.

We focus on the decision problem facing a *focal bidder at his margin*, that is, when (a) the current bidding level is one bid increment away from the bidder's maximum feasible bid, (b) the auction has not ended, and (c) the bidder has a chance to submit a (new) bid. When a bidder arrives at his margin, he has two options: (i) following a participatory strategy to bid the current bidding level $b = B - \delta$, which lands him at the x-th position on the provisional winning list or (ii) following a jump strategy by bidding his maximal feasible bid, that is, to bid B, which hands him at the y-th position. Naturally, we have $y \leq x$. In particular, if the bidder never arrives at his margin, he will keep using *participatory* strategy throughout the auction.

We assume there are no bidding cost. A bidder's payoff, if he wins, is equal to the difference between his valuation and his winning bid. If he wins the auction using a participatory strategy at his margin, his payoff is denoted by U_p ; If he wins using a jump strategy, U_j. We also assume that m bidders have maximum bids of $B - \delta$ or higher and n $(n \leq m)$ bidders have maximum bids of B or higher. We assume $m \geq Q$ and $n \geq Q$ to avoid the trivial cases. We illustrate our model setting by the following example:

Example 1: Suppose there are $N = 10$ bidders competing for $Q = 5$ identical units. Both the opening bid O and bid increment δ are 1. Let the 10 bidder's valuations be $\{V_i\} = \{\mathbf{5}, 4, 4, 4, 4, 3, 2, 2, 2, 2\}$. We further assume that only the first bidder has a budget constraint of $B = 4$ (hence his budget gap is $D = 1$). So the maximum bids are $\{B_i\} = \{\mathbf{4}, 4, 4, 4, 4, 3, 2, 2, 2, 2\}$. Let the first bidder be our focal bidder. We can see that $m = 6$ bidders can bid at the focal bidder's margin $B - \delta = 3$ and $n = 5$ bidders who can bid one increment higher, at $B = 4$. The winning level of the auction is $W = 3$. Suppose the focal bidder arrives at his margin with a provisional winning list of $\{3, 3, 3, 3, 2\}$. The bidding level, by definition, is $b = 3$. For the focal bidder, there is only one increment $(\delta = 1)$ away from the required bidding level to his feasible bid. He has two options for his bidding strategy: (i) he could follow a participatory strategy to bid $B - \delta = 3$, which lands him at the 5-th $(x = 5)$ position. The new winning list would be $\{3, 3, 3, 3, \mathbf{3}\}$. Alternatively, (ii) he could follow a jump strategy to bid at level $B = 4$, which lands him the first position $(y = 1)$ in the winning list. The new winning list would be $\{\mathbf{4}, 3, 3, 3, 3\}$.

We summarize our notations in Table 1.

Notation	*Interpretation*
N	Number of bidders who participate in the auction
Q	Number of identical units for sale
δ	Minimal bid increment
O	Opening bid of the auction
W	Winning level of the auction
F	Cumulative density function of all bidders' valuations
V_i	Valuation of bidder i
B_i	Maximum feasible bid of bidder i
G_i	Budget gap of bidder i
m	Number of bidders (excluding the focal bidder) who can bid $B-\delta$ or higher
n	Number of bidders (excluding the focal bidder) who can bid B or higher
x	The position on the winning list when the focal bidder bids $B-\delta$ at his margin
y	The position on the winning list when the focal bidder bids B at his margin
U_p	Payoff of the focal bidder when he uses a *participatory* strategy at his margin
U_j	Payoff of the focal bidder when he uses a *jump* strategy at his margin

Table 1. Notations

Optimal Bidding Strategy at the Margin

Example 2: Continue with Example 1. (i) Suppose the auction arrives at the focal bidder's margin, whether he can win the auction at this bidding level depends on how many bidders, other than bidders already on the winning list, can bid higher than 3. If the four bidders whose maximum bid is 4 are on the current winning list, the focal bidder will win at 3, which means he pays a price of 3 with a payoff of $5-3=2$. However, if the bidder at the 4-th position has a maximum bid of 3, which means that a bidder with a maximum bid of 4 will eventually displace the focal bidder with a bid of 4. Then the focal bidder will rebid at the level 4 to enter the winning list. He will finally win at price of 4 with a payoff of $5-4=1$. (ii) Now suppose the focal bidder instead follows a jump strategy to bid $B=4$ at his margin. He will win with certainty with a payoff of 1.

From example 2, we can see the *participatory* strategy yields a higher surplus than the *jump* strategy while maintaining the winning probability. So, in this case, the *participatory* strategy outperforms the *jump* strategy. However, the *participatory* strategy is not always optimal, as seen in the next example.

Example 3: Continue with Examples 1 and 2. In this case, suppose there are $n=6$ bidders who have a maximal bid of 4, the maximum bids are $\{B_i\}=\{\mathbf{4},4,4,4,4,4,3,2,2,2\}$. The winning level $W=4$, which means the focal bidder cannot win at the price 3. When the auction arrives at his margin (as in Example 1, we assume the provision winning bids are $\{3,3,3,3,2\}$), if he chooses a *participatory* strategy to bid 3, he may not have a chance to rebid (if five other bidders bid 4 before him), which results in him losing the auction and earning a zero payoff. If he wins, he submit a rebid of 4, yielding a payoff of 5-4=1. In contrast, if he chooses a *jump* strategy to bid 4, he is the first to bid 4 and will win the auction for sure, with a payoff of 5-4=1. So, in this case, *jump* is superior to *participatory*.

From the above examples, we can see that the optimal strategy for a budget-constrained bidder depends on several factors. Next, we analyze the payoff of each strategy, and derive the optimal bidding strategy at the margin. The sequence of events and decisions for different strategies is captured in the following decision tree in Figure 1.

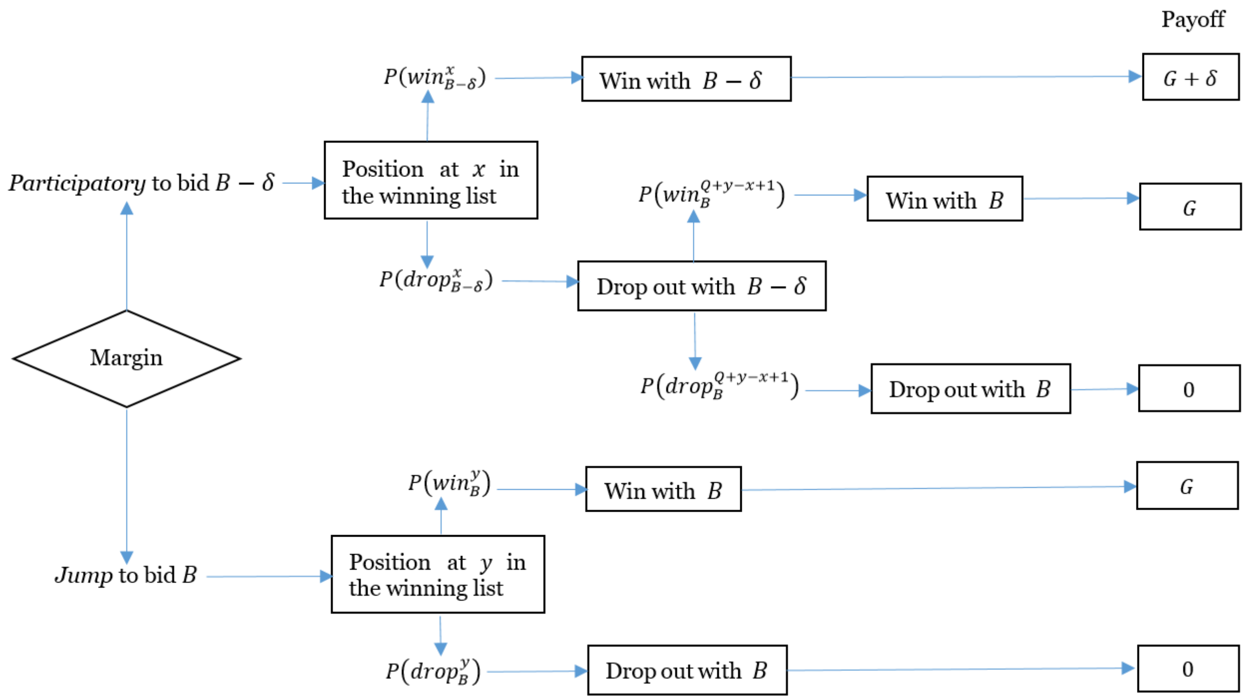

Figure 1. Decision Tree for a Budget-constrained Bidder at His Margin

Participatory Strategy

When the bidder employs a participatory strategy at the margin, he may either (a) win with a bid of $B-\delta$, which yields a payoff of $V-(B-\delta)=V-B+\delta=G+\delta$, or (b) lose with $B-\delta$ but win with a revised bid of B, which yields a payoff of G. We examine these two cases in turn.

Denote his probability of winning, when he bids $B-\delta$ and earns the x-th position on the current winning list, as $P(win^x_{B-\delta})$. When the bidder loses with a bid of $B-\delta$ (which places him at the x-th position), it means at least $Q-(x-1)$ bidders enter the winning list and rank higher than him, causing him to be displaced. In other words, at least $Q-(x-1)$ bidders not on the current winning list can bid greater than or equal to B. For the focal bidder, the probability of dropping out and rebidding B, denoted by $P(drop^x_{B-\delta})$, is given by.

$$P(drop^x_{B-\delta}) = C_m^{x-1}\left[\sum_{r=Q-(x-1)}^{m-(x-1)} C^r_{m-(x-1)}\big(1-F(B)\big)^r\big(F(B)\big)^{m-(x-1)-r}\right] \quad (1)$$

In equation (1) and subsequently, the notation C is for combinations. Equation 1 captures the probability that among *m-(x-1)* bidders who can bid $B-\delta$ or higher and are not on the current winning list, at least $Q-(x-1)$ and at most $m-(x-1)$ can bid B. We then have:

$$P(win^x_{B-\delta}) = 1 - P(drop^x_{B-\delta}) \quad (2)$$

We now consider the case (b). If the bidder rebids B after dropping out, the best position he can get is $y+(Q-x+1)=Q+y-x+1$ (noting that he would have obtained y when he bids B at the margin, but with the participatory strategy, at least $(Q-x+1)$ bidders bid B ahead of him). The probability of winning with a bid of B when positioned at $Q+y-x+1$ is denoted by $P\big(win_B^{Q+y-x+1}\big)$. Similar to the analysis for (1), if he loses in such a case, it means that at least $Q-(Q+y-x)=(x-y)$ bidders who are not on the current winning list can bid greater than or equal to $B+\delta$. Once he drops out at this level, he will lose the auction for sure and earn a zero payoff. Therefore, we have

$$P\big(drop_B^{Q+y-x+1}\big) = C_n^{Q+y-x}\left[\sum_{r=x-y}^{n-(Q+y-x)} C^r_{n-(Q+y-x)}\big(1-F(B+\delta)\big)^r\big(F(B+\delta)\big)^{n-(Q+y-x)-r}\right] \quad (3)$$

Equation (3) captures the probability that among $n-(Q+y-x)$ bidders who can bid B or higher and are not on the current winning list, at least $x-y$ can bid $B+\delta$. And we also will have:

$$P\left(win_B^{Q+y-x+1}\right) = 1 - P\left(drop_B^{Q+y-x+1}\right) \tag{4}$$

Given the above, the expected payoff for a budget-constrained bidder who employs a participatory strategy at his margin is:

$$E\left[U_p\right] = P(win_{B-\delta}^x)(G+\delta) + P(drop_{B-\delta}^x)P\left(win_B^{Q+y-x+1}\right)G \tag{5}$$

Jump Strategy

When the budget-constrained bidder employs a jump strategy at his margin, he bids B and earns position y. Denote his probability of winning the auction, given his bid and position, as $P\left(win_B^y\right)$. If he wins with such a bid, his payoff is $V-B=G$.

When the bidder loses, it means at least $Q-(y-1)$ bidders enter the winning list after him with a higher bid. IN other words, $Q-(y-1)$ bidders not on the current winning list can bid greater than or equal to $B+\delta$. Hence

$$P\left(drop_B^y\right) = C_n^{y-1}\left[\sum_{r=Q-(y-1)}^{n-(y-1)} C_{n-(y-1)}^r \left(1-F(B+\delta)\right)^r \left(F(B+\delta)\right)^{n-(y-1)-r}\right] \tag{6}$$

Equation (6) captures the probability that among $n-(y-1)$ bidders who can bid B or higher and are not on the current winning list, at least $Q-(y-1)$ can bid $B+\delta$. We also have:

$$P\left(win_B^y\right) = 1 - P\left(drop_B^y\right) \tag{7}$$

So the expected payoff for a budget-constrained bidder who employs a jump strategy at his margin is:

$$E\left[U_j\right] = P\left(win_B^y\right)G \tag{8}$$

Optimum Jumping

If the expected surplus of the jump strategy is higher than that of the participatory strategy, he will jump, otherwise, he will play a participatory strategy. The condition for *jump* to be optimal is:

$$E\left[U_j\right] > E\left[U_p\right] \tag{9}$$

Plug equations (5) and (8) to equation (9), we have:

$$P\left(win_B^y\right)G > P(win_{B-\delta}^x)(G+\delta) + P(drop_{B-\delta}^x)P\left(win_B^{Q+y-x+1}\right)G \tag{10}$$

Then, we can get:

$$\frac{G}{G+\delta} > \frac{P\left(win_{B-\delta}^x\right)}{P\left(win_B^y\right) - P\left(drop_{B-\delta}^x\right)P\left(win_B^{Q+y-x+1}\right)} \tag{11}$$

We define:

$$A = \frac{P\left(win_{B-\delta}^x\right)}{P\left(win_B^y\right) - P\left(drop_{B-\delta}^x\right) * P\left(win_B^{Q+y-x+1}\right)} \tag{12}$$

Equation (11) can be rewritten as,

$$\frac{G}{\delta} > \frac{A}{1-A} \tag{13}$$

PROPOSITION 1. *When a budget-constrained bidder comes to his margin during the auction, the bidder should play a jump strategy to bid his maximum feasible bid if the condition (13) holds; otherwise he should play a participatory strategy to bid the current minimum required level.*

If a bidder plays his strategy according to Proposition 1, we call such a bidder a strategic-at-margin (SAM) bidder.

Discrete Event Simulation (DES)

The first objective of simulation is to validate our proposed strategic-at-margin (SAM) bidding strategy by benchmarking it against the participatory and jump bidding strategies. Our second objective is to investigate the key factors driving the SAM threshold for jumping. Based on the theoretical analysis, the budget gap and the bid increment jointly influence the SAM threshold. So in the simulation, we perturb budget and bid increment values and study their effects.

In this simulation, we choose one bidder as the focal bidder to compete with the remaining bidders. The focal bidder follows one of the three bidding strategies (Participatory, Jump, and SAM) while others use the Participatory strategy. The three types of bidding strategies are described as follows:

- *Participatory*: always place the minimum required bid throughout the auction.
- *Jump*: behave like participatory bidders except that they will place a jump bid (i.e., one increment above the minimum required bid) at the margin.
- *SAM*: behave like participatory bidders except that they will place a jump or a participatory bid according to Proposition 1 at the margin.

In the simulations, we draw bidders' valuations from a uniformly distribution. The sequence of arrival is controlled by a random number stream, which vary for each simulation iteration. We randomly choose a bidder as the focal bidder, provided that his valuation is higher than the auction's winning level. Only the focal bidder has a budget constraint (it is trivial to extend to cases where other bidders also have a budget constraint by simply replacing their valuations with their budgets).

The simulation is implemented using Java. For each parameter set, we compare the performance of the focal bidder adopting SAM vs. two other bidding strategies. We use two measurements for comparison: (1) the likelihood of winning and (2) the average payoff the bidder gets. For each parameter set and bidding strategy, to get statistically robust results, we conduct 30 repetitions varying only the random number stream (thus the arrival sequence) while keeping all other aspects the same. We note that, due to different arrival sequences, the focal bidder may arrive at the margin with a different winning list, which will result in different $\frac{A}{1-A}$ values. So the SAM strategy as well as two other strategies will yield different outcomes across repetitions.

Some parameters including the number of bidders, the number of units, the distribution of valuations, and the opening bid are fixed (see Table 2 for these values). The randomly-drawn valuation of the focal bidder is 98.

N	Q	Lower bound of V_i	Upper bound of V_i	O
50	10	50	100	5

Table 2. Fixed Parameters of Simulations

Simulation Results

SAM vs. Participatory and Jump

We compare the performance in terms of likelihood of winning and payoff between *SAM* and two other bidding strategies. When we perturb budget and bid increment values in the following Table 3, Table 6 as well as Table 7, we use the same set of valuation distribution.

$\boldsymbol{B=90}$, $\delta=5$, $W=85$, $\boldsymbol{G/\delta=1.60}$						
Variables	Likelihood of winning			Payoff		
	SAM	*Participatory*	*Jump*	*SAM*	*Participatory*	*Jump*
Obs	30	30	30	30	30	30
Mean (SD)	1.00 (0)	1.00(0)	1.00(0)	9.19(2.15)	9.19(2.15)	8.01(0)

t test data	(a): $t(29)$=n.a. , p = n.a. (b): $t(29)$=n.a. , p = n.a.	(a): $t(29)$=n.a. , p = n.a. (b): $\boldsymbol{t(29)=2.97}$, $p < 0.01$
	(a): *SAM = Participatory;* (b) *SAM = Jump*	

Table 3a. SAM vs. Participatory and Jump under $(\boldsymbol{budget}, \boldsymbol{increment}) = (\mathbf{90}, \mathbf{5})$

$\boldsymbol{B = 85}$, $\delta = 5$, $W = 85$, $\boldsymbol{G/\delta = 2.60}$						
Variables	Likelihood of winning			Payoff		
	SAM	*Participatory*	*Jump*	*SAM*	*Participatory*	*Jump*
Obs	30	30	30	30	30	30
Mean (SD)	0.70(0.47)	0.33(0.48)	0.73(0.45)	9.11(6.07)	4.34(2.01)	9.55(5.86)
t test data	(a): $\boldsymbol{t(29)=3.27}$, $p < 0.01$ (b): $t(29)$=1.00 , p = 0.33			(a): $\boldsymbol{t(29)=3.27}$, $p < 0.01$ (b): $t(29)$=1.00 , p = 0.33		
	(a): *SAM = Participatory;* (b) *SAM = Jump*					

Table 3b. SAM vs. Participatory and Jump under $(\boldsymbol{budget}, \boldsymbol{increment}) = (\mathbf{85}, \mathbf{5})$

$B = 90$, $\boldsymbol{\delta = 3}$, $\boldsymbol{W = 89}$, $\boldsymbol{G/\delta = 2.67}$				
Variables	Likelihood of winning		Payoff	
	SAM	*Participatory*	*SAM*	*Participatory*
Obs	30	30	30	30
Mean (SD)	0.70(0.47)	0.5(0.51)	6.31(4.20)	4.51(4.59)
t test data	$\boldsymbol{t(29)=2.69}$, p=0.01		$\boldsymbol{t(29)=2.69}$, p=0.01	

Table 3c. SAM vs. Participatory under $(\boldsymbol{budget}, \boldsymbol{increment}) = (\mathbf{90}, \mathbf{3})$

From Table 3a, first, we can see *SAM* has the same performance as *participatory*. In other words, when G/δ ratio = 1.60, the SAM bidder never jumps and places only *participatory* bids at the margin. Comparing Table 3a with Table 3b, we note that as the budget gap (and hence the G/δ ratio) increases, the bidder more likely wins using the *SAM* strategy and obtains a higher payoff than the *participatory* strategy.

Comparing Table 3a with Table 3c, we note that when the bid increment deceases, the *SAM* strategy outperforms the *participatory* strategy.

Based on the above analysis, we can confirm that overall, *SAM* outperforms *participatory*. Moreover, as budget gap is becoming larger and bid increment becoming smaller, the G/δ ratio increases, then there is a higher likelihood for condition (13) to hold, which means the bidder is more likely to jump instead of keeping participatory at his margin. The strategy becomes a more advantageous one: *SAM*.

Then, we compare the performance between *SAM* and *jump* strategies.

From Table 3a, when G/δ ratio = 1.60, there is no difference in likelihood of winning between *SAM* and *jump* strategies. *SAM's* payoff is however higher than *jump*, which means *SAM* can win the auction in lower prices. Overall, *SAM* outperforms *jump*. By comparing Table 3a and Table 3b, we find that as G/δ ratio increases, there is no significant difference in either the likelihood of winning or the payoff between *SAM* and *jump* strategies.

Robustness checks

From above simulations, we observe that *SAM* outperforms the other two strategies for a specific set of valuations. To further test the robustness of our findings, we conduct simulations under different realization of valuations. The pair of ($budget, increment$) is fixed at (90,5) for different sets of valuations. To make sure that G/δ ratio doesn't change, we fix the valuation of the focal bidder at 98. We generate 30 different sets of uniformly distributed valuations for the other 49 bidders. In each set of valuations, we conduct 30 simulations as before, resulting in 30*30 simulations. For each set of valuations, we record the likelihood of winning and the average payoff for *SAM, participatory* and *jump* strategies, respectively (Table 4).

Valuation Set	Likelihood of winning			Average payoff		
	SAM	*Participatory*	*Jump*	*SAM*	*Participatory*	*Jump*
1	0.87	0.87	1.00	6.95	6.95	8.02
2	1.00	1.00	1.00	9.85	9.85	8.02
3	1.00	1.00	1.00	11.85	11.85	8.02
4	0.90	0.67	0.97	7.22	5.35	7.75
5	1.00	1.00	1.00	11.19	11.19	8.02
6	1.00	1.00	1.00	9.69	9.69	8.02
7	0.80	0.67	0.90	6.41	5.35	7.22
8	1.00	1.00	1.00	11.52	11.52	8.02
9	1.00	1.00	1.00	8.02	8.02	8.02
10	0.87	0.57	0.87	6.95	4.54	6.95
11	0.63	0.43	0.73	5.08	3.47	5.88
12	1.00	1.00	1.00	8.35	8.35	8.02
13	0.60	0.53	0.90	4.81	4.28	7.22
14	1.00	1.00	1.00	10.19	10.19	8.02
15	0.80	0.80	1.00	6.41	6.41	8.02
16	1.00	1.00	1.00	9.69	9.69	8.02
17	1.00	1.00	1.00	9.35	9.35	8.02
18	0.73	0.70	1.00	5.88	5.61	8.02
19	1.00	1.00	1.00	12.69	12.69	8.02
20	0.47	0.17	0.50	3.74	1.34	4.01
21	0.70	0.57	0.87	5.61	4.54	6.95
22	1.00	1.00	1.00	10.19	10.19	8.02
23	1.00	1.00	1.00	9.85	9.85	8.02
24	0.93	0.63	0.93	7.48	5.08	7.48
25	1.00	1.00	1.00	10.52	10.52	8.02
26	1.00	1.00	1.00	9.35	9.35	8.02
27	1.00	1.00	1.00	9.69	9.69	8.02
28	0.63	0.23	0.80	5.08	1.87	6.41
29	0.83	0.83	1.00	6.68	6.68	8.02
30	0.80	0.60	0.83	6.41	4.81	6.68

Table 4. The Performances of the Three Strategies under 30 Sets of Valuations

$B = 90,\ \delta = 5,\ W = 85,\ G/\delta = 1.60$						
Variables	Likelihood of winning			Payoff		
	SAM	*Participatory*	*Jump*	*SAM*	*Participatory*	*Jump*
Obs	30	30	30	30	30	30
Mean (SD)	0.89(0.15)	0.81(0.25)	0.94(0.11)	8.22(2.36)	7.61(3.07)	7.57(0.89)
t test data	(a):$\boldsymbol{t(29)=3.44}$, $p = 0.002$ (b):$\boldsymbol{t(29)=3.55}$, $p = 0.001$ (c):$\boldsymbol{t(29)=4.54}$, $p < 0.001$			(a):$\boldsymbol{t(29)=3.45}$, $p < 0.002$ (b):$\boldsymbol{t(29)=1.93}$, $p = 0.03$ (c):$t(29)=0.10$, $p = 0.92$		
	(a): *SAM* = *Participatory;* (b) *SAM* = *Jump;* (c) *Participatory* = *Jump*					

Table 5. Likelihood of Winning and Average Payoff for the Three Strategies under 30 Sets of Valuations

From Table 5, we can conclude that *SAM* has a higher likelihood of winning the auction than *participatory* but a lower likelihood than *jump*. We also find that *SAM* has a higher payoff than *participatory* and *jump*. Although *jump* has a higher winning likelihood than *SAM*, its payoff is lower. Overall, *SAM* is optimal among the three. Figure 2 shows the performance comparison among the three bidding strategies.

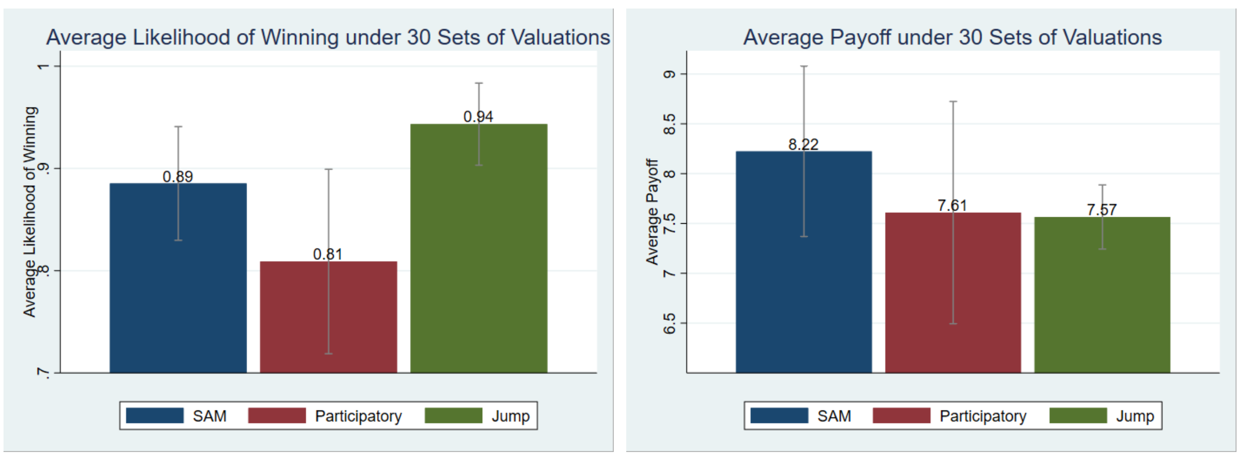

Figure 2. Performance Comparison Among the Three Bidding Strategies

Proportional SAM

From the above analysis, we conclude that *SAM* is better performing than other strategies for a budget-constrained bidder. However, to carry out *SAM*, the focal bidder needs complete information about other bidders' valuations. In reality, the bidder may not have such information and may have information on the bids in winning lists. Bapna et al. investigate the performance of proportional jump bidding strategy which is to jump bid in proportion to the number of bidders who are in the winning lists with bids at current required bid or higher(Bapna, Goes, & Gupta, 2003). For example, in a five-unit ascending auction with bid increment of 1, the current minimum required bid is 4. If the bidder observes that there are two bids at level 4 or higher in the winning list, he will bid 5 with a probability of 2/5 as well as 4 with a probability of 3/5. They find that this proportional jump bidding doesn't increase the likelihood of winning while has a lower payoff than *participatory* strategy. However, in that study, they neither consider a budget constraint nor focus the proportional jump bidding at the bidder's margin.

To mitigate the above issue, we propose and examine a new proportional jump bidding strategy at a budget-constrained bidder's margin, which is called *proportional SAM* (*pSAM*). When a budget-constrained bidder comes to his margin, the current minimum required bid is $B - \delta$. Suppose there are k bids at level $B - \delta$ or higher in the winning list. If the proportion k/Q (Q is number of units) is higher than the bidder's expected value, he will jump by one increment to bid B, otherwise, he will bid $B - \delta$. As we don't know the focal bidder's expected cutoff proportion, we use a probability method here, that is, the bidder will bid B with a probability of k/Q and $B - \delta$ with a probability of $(1 - \frac{k}{Q})$.

The setup of simulation is the same as before. We compare the performance in terms of the likelihood of winning and expected payoff between *pSAM* and other three strategies, which are *SAM, participatory, and jump,* under the pair of $(budget, increment)$: $(85,5)$. We also conduct 30 repetitions in one set of valuation.

$B = 85,\ \delta = 5,\ W = 85,\ G/\delta = 2.60$				
Variables	Likelihood of Winning			
	pSAM	*Participatory*	*Jump*	*SAM*
Obs	30	30	30	30
Mean(SD)	0.50(0.59)	0.33(0.48)	0.73(0.45)	0.70(0.47)
t test data	(a):$\boldsymbol{t}$**(29)=3.00,** p = 0.01 (b):t(29)=1.71, p = 0.10 (c):t(29)=1.44, p =0.16			
	(a): *pSAM = Participatory;* (b) *pSAM = Jump;* (c) *pSAM = SAM*			

Table 6a. Likelihood of Winning for Proportional SAM vs. Other Three Strategies under $(\boldsymbol{budget}, \boldsymbol{increment}) = (\mathbf{85}, \mathbf{5})$

$B = 85,\ \delta = 5,\ W = 85,\ G/\delta = 2.60$				
Variables	Likelihood of Winning			
	pSAM	*Participatory*	*Jump*	*SAM*
Obs	30	30	30	30
Mean(SD)	6.51(7.65)	4.34(6.24)	9.55(5.86)	9.11(6.07)
t test data	(a):$\boldsymbol{t}$**(29)=3.00,** p = 0.01 (b):t(29)=1.71, p = 0.10 (c):t(29)=1.44, p =0.16			
	(a): *pSAM = Participatory;* (b) *pSAM = Jump;* (c) *pSAM = SAM*			

Table 6b. Payoffs for Proportional SAM vs. Other Three Strategies under $(\boldsymbol{budget}, \boldsymbol{increment}) = (\mathbf{85}, \mathbf{5})$

From Table 6a and Table 6b, *proportional SAM* has a higher likelihood of winning and yields a higher payoff than the *participatory* strategy. For comparison with *jump and theoretical SAM*, there are no significant differences. Overall, the *proportional SAM* is advantageous.

Conclusions and Future work

In this paper, we investigate the role of the budget constraint as a reason for jump bidding in multi-unit ascending online auctions. We theoretically derive the conditions under which jump bidding outperforms the participatory strategy for bidders at their margin.

We conclude that the budget gap and the bid increment *jointly* influence the bidding strategy for the budget-constrained bidder at his margin (i.e. one increment away from their maximum bid). Based on theoretical analysis, we propose a hybrid strategic-at-margin (*SAM*) bidding strategy. Then we use a simulation model to validate our proposed *SAM* strategy can outperform *participatory* and *jump* strategies and to examine how the budget gap and the bid increment influence the bidding strategy.

The findings of this study are: (i) Budget constraints can motivate bidders to submit jump bids at their margin. This driver of jump bidding does not depend on participation costs or bidder impatience and is in force even in an agent-based bidding environment. (ii) The optimal strategy for a budget-constrained at-the-margin bidder is a hybrid one, which is the *SAM* bidding developed in this paper. (iii) Jump bidding increases with the budget gap (i.e. the difference between a bidder's budget and his valuation) and decreases with the bid increment. In addition, we also find some initial evidence that *proportional SAM* bidding, which eases the information requirement on bidders, outperforms a participatory strategy.

As a preliminary study on budget-induced jump bidding, there are several future directions. First, we can examine the robustness of the strategy when a bidder's opponent can also adopt the proposed *SAM* strategy. Second, we plan to compare more heuristic strategies that operate well under incomplete information. Third, a lab experiment can be conducted to further validate and evaluate the proposed strategy in more realistic settings.

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